Using a Directional Wattmeter to measure power in a mismatched transmission line

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Abstract

This article presents a derivation of the power at a point in a mismatched transmission line in terms of ρ, the magnitude of the complex reflection coefficient Γ, and Forward Power and Reflected Power as might be indicated by a Directional Wattmeter such as a Bird 43. Mismatch Loss is also explained.

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1. Power in a mismatched transmission line

We start by deriving the Apparent Power P_a in terms of transmission line parameters.

$P_a = V.I^*$

Γ is the complex reflection coefficient, let $\Gamma = a + jb$, and let ρ be the magnitude of Γ, so that $\rho^2 = |\Gamma|^2 = a^2 + b^2$. V_f, V_r, I_f, and I_r are the forward and reflected (or reverse) components of V and I, and Z_0 is the characteristic impedance of the transmission line.

Now, solving P_a in terms of the above...

$V = (1 + \Gamma).V_f$

$V = (1 + a + jb).V_f$

$I = (1 - \Gamma).I_f$

$I = (1 - a - jb).I_f$

Since $I_f = \frac{V_f}{Z_0}$...

$$I = (1 - a - jb).\frac{V_f}{Z_0}$$

$$P_a = (1 + a + jb).V_f.(1 - a - jb).\left(\frac{V_f}{Z_0}\right)^*$$

$$P_a = \frac{|V_f|^2}{{Z_0}^*}\big((1 + a)(1 - a) + j(1 - a)b + j(1 + a)b - b^2\big)$$

P_a is a complex quantity having both real and imaginary components. We are only interested in the real component of P_a, which we can write as P_r.

2. Simplification for Distortionless Lines

In the case where Z_0 is real, meaning purely resistive, (Distortionless Lines, which includes Lossless Lines), the real power P_r can be simplified to ...

$$P_r = \frac{|V_f|^2}{Z_0}\left(1 - (a^2 + b^2)\right)$$

$$P_r = \frac{|V_f|^2}{Z_0}(1 - \rho^2)$$

Since $|V_r| = |\Gamma|.|V_f| = \rho.|V_f|$...

$$P_r = \frac{|V_f|^2}{Z_0} - \frac{|V_r|^2}{Z_0}$$

The quantity $\frac{|V_f|^2}{Z_0}$ is commonly known as P_{fwd} , and $\frac{|V_r|^2}{Z_0}$ is commonly known as P_{ref} and are the quantities indicated by common Directional Wattmeters. So ...

$$P_r = \frac{|V_f|^2}{Z_0} - \frac{|V_r|^2}{Z_0} = P_{fwd} - P_{ref}$$

Note that $P_r = P_{fwd} - P_{ref}$ is true only when Z_0 is real, and such instruments are usually calibrated for a real Z_0.

3. Mismatch Loss

Mismatch Loss when applied to transmission lines is the ratio of incident power to the power accepted by the load.

Since for distortionless lines $P_{incident} = \frac{|V_f|^2}{Z_0}$ and $P_{load} = \frac{|V_f|^2}{Z_0}(1-\rho^2)$, Mismatch Loss is easily calculated for that case.

$$MismatchLoss = \frac{P_{incident}}{P_{load}} = \frac{\frac{|V_f|^2}{Z_0}}{\frac{|V_f|^2}{Z_0}(1-\rho^2)}$$

Mismatch Loss can be calculated in dB as $MismatchLoss = -10\log(1-\rho^2)$, and since $\rho = \frac{\text{VSWR}-1}{\text{VSWR}+1}$,
$MismatchLoss = -10\log\left(1-\left(\frac{\text{VSWR}-1}{\text{VSWR}+1}\right)^2\right)$.

For example, if VSWR=2, Mismatch Loss is 0.5dB.

Cascaded mismatches are more difficult to calculate, it is not valid to simply add the dB Mismatch Loss of cascaded mismatches. Simple formulas that do not fully account for transmission line effects may have significant error.

For example, consider an electrical half wave of lossless 75Ω coax inserted in a matched lossless 50Ω system (meaning 50+j0Ω source, lossless 50+j0Ω line and 50+j0Ω load). By definition, of Mismatch Loss above, the Mismatch Loss at the load end of the 75Ω line section with 50+j0Ω load is 0.18dB. There is another possible mismatch at the source end of the 75Ω line section and Mismatch Loss is 0dB (by virtue of the 50+j0Ω load load resulting from the impedance transformation in the 75Ω line section). In fact the combined effect is 0dB loss.

In the real world, instruments tend to be calibrated wrt real Z (eg 50+j0Ω) so their measurements comply with the real Z_0 assumption, but practical transmission lines have Z_0 that is not real. though the error would usually be insignificant above 30MHz.

Mismatch Loss is widely misused. If the underlying criteria are not satisfied, the basis for the calculation does not exist and the figure obtained is invalid. For example, it is an error to infer the reduced output from a transmitter into a given VSWR(50) load using the formula above unless you know that the transmitter behaves as a linear system with equivalent source impedance 50+j0Ω so that there is only a single mismatch of known character. (Note, a transmitter designed for a load of 50+j0Ω need not, and probably won't have an equivalent source impedance of 50+j0Ω.)

The definition of Mismatch Loss says nothing about the implementation of the source or any loss or dissipation occurring within it. Mismatch could result in higher or lower internal loss in the source, depending on implementation.

4. Practical power measurement

It is the impedance Z_0 for which the Directional Wattmeter is calibrated that is important, not the Z_0 of the transmission line external to the instrument, or even its internal line to a certain extent (neither of which are perfectly real). If the Directional Wattmeter's sampling element is calibrated for zero P_{ref} on a practical resistive termination at the end of a very short transmission line, the error in assuming that $P_r = P_{fwd} - P_{ref}$ is very small, insignificant wrt typical standard error of RF power measurement.

For example, if a Bird 43 calibrated for 50Ω is inserted in a 75Ω line, P_{fwd} reads 100W and P_{ref} reads 10W, the power (ie rate at which energy flows past that point) is 90W.

Obviously, as P_{ref} approaches P_{fwd}, the standard error of the calculated power increases, and the technique is of limited use in extreme VSWR cases.

If a Bird 43 is used to measure the output of a nominally 100W transmitter, and P_{fwd} reads 109W and P_{ref} reads 4.4W, the power (ie rate at which energy flows past that point) is 104.6W and VSWR(50)=1.5. It would be an error to apply Mismatch Loss for VSWR(50)=1.5 to the 100W nominal transmitter output rating to calculate power output under mismatch as actual forward power may be higher or lower than nominal for several reasons, some that are load dependent, including that the equivalent source impedance may not be a constant 50+j0Ω.

5. Conclusions

- Mismatch Loss, $MismatchLoss = -10\log(1-\rho^2)$, is the reduction of power in a load due to mismatch when:
  - Z_0 is real (purely resistive);
  - the equivalent source impedance is equal to Z_0; and
  - it implies nothing about dissipation within source.
- A Directional Wattmeter can be used to determine power in a transmission line:
  - if the calibration impedance is real (ie purely resistive);
  - even if the impedance $(\frac{V}{I})$ at the point it is sampling is not equal to its calibration impedance;
  - a known resistive load is not necessary; and
  - power equals Forward Power less Reflected Power.