Calculation of Matched Line Loss from measurement of Zin at resonance or antiresonance of a short circuit or open circuit transmission line section

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Abstract

This article presents a derivation of an expression for calculation of Matched Line Loss from measurement of Zin at resonance or antiresonance of a short circuit or open circuit transmission line section.

1. Derivation of Matched Line Loss (per unit length) from measurement of Zin at resonance or antiresonance of a short circuit or open circuit line section.

An expression for the input impedance of a transmission line section with a given load can be derived from the Telegrapher's Equation.

$$Z_{in} = Z_0 \frac{Z_l + Z_0 tanh(\gamma l)}{Z_0 + Z_l tanh(\gamma l)}$$

When the line termination is a short circuit, $\frac{Z_{in}}{Z_0} = \tanh(\gamma l)$ where $\gamma = \alpha + j\beta$. Zin will be minimum and $R_{in} \cong |Z_{in}|$ at resonance where l is equivalent to an even number of electrical quarter waves.

Since $\tanh(x) = \frac{1-e^{-2x}}{1+e^{-2x}}$, we can say $\frac{Z_{in}}{Z_0} = \frac{1-e^{-2\gamma l}}{1+e^{-2\gamma l}}$,which leads to $e^{-2\gamma l} = \frac{Z_0 - Z_{in}}{Z_0 + Z_{in}}$ and $\gamma = \frac{-1}{2l} ln\left(\frac{Z_0 - Z_{in}}{Z_0 + Z_{in}}\right)$

Since l is chosen so that $\beta \cong 0$, $\alpha = \frac{-1}{2l} ln\left(\frac{Z_0 - Z_{in}}{Z_0 + Z_{in}}\right)$ and converting from Nepers to dB, $MLL = \frac{-10}{l} log\left(\frac{Z_0 - Z_{in}}{Z_0 + Z_{in}}\right)$

When the expressions are developed for the cases of open circuit and short circuit terminations, each with resonant and antiresonant lengths, the expressions can be simplified to a single expression that is easy to calculate.

$$MLL = \frac{-10}{l} log\left(\left|\frac{Z_0 - Z_{in}}{Z_0 + Z_{in}}\right|\right)$$

2. An approximation

As x approaches zero, $tanh(x) \cong x$ and vice versa with error of <1% for $x < 0.17$, so the expressions above can be simplified to $MLL = \frac{8.686 Z_{in}}{Z_0 . l}$ at resonance and $MLL = \frac{8.686 Z_0}{Z_{in} . l}$ at antiresonance.

This approximation if often given without declaring that it is only an approximation, and the limits of accuracy.

3. Uncertainty

The simplified approach assumes Z_0 is the nominal real value given to the transmission line. The results are sensitive to error in Z_0 due to manufacturing tolerances etc. Ignoring the reactive component of Z_0 should make insignificant difference in most practical low loss transmission lines above 10MHz, and very little difference above 1MHz.

Uncertainty in measurement of Z_{in} will usually dominate total uncertainty. R_{in} changes slowly about resonance and antiresonance, so whilst frequency sensitive, it is not unduly so and most of the uncertainty is that of RF impedance measurement. Selection of an appropriate instrument and test scenario is important to accuracy.

4. References

•Duffy. Jan 2014. Measuring matched line loss. http://owenduffy.net/blog/?p=310.

•———. Nov 2007. RG−6/U for transmitting applications. VK1OD.net (offline).