

Uncertainty of the noise sampling process

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Abstract

This article examines the uncertainty of the process of sampling noise for measurement, and a method for prediction of sampling uncertainty. The proposed solution has application generally to high resolution measurement of bandwidth limited noise.

1. Characteristics of noise

Broadband noise is caused by a random process that generates a voltage that varies randomly over time.

The noise voltage is a normally distributed random variable.

Sampling is the process of converting a signal (for example, a function of continuous time or space) into a regular numeric sequence (a function of discrete time or space).

The noise voltage can be sampled, and the instantaneous power is proportional to the square of the sample value.

The average noise power can be estimated by averaging the instantaneous power at regular intervals over a period of time. $P_e = \frac{\sum(v_i)^2}{nR}$ where P_e is the estimated noise power in a resistor, v_i are the voltage samples, n is the number of samples and R is the load resistance.

It is important to remember that the measurement process is only an estimate of the true noise power, and that successive sample sets are likely to produce similar, but different estimates.

An explanation about the probability distribution of this sampling process can be obtained from classic statistics.

Statistically, the noise voltage samples are normally distributed, have a mean of zero, and some variance. The RMS voltage is equal to the square root of the variance.

If x_i samples of an independent normally distributed random variable with mean 0 and variance 1, then the random variable $Q = \sum x_i^2$ is distributed according to the chi-square distribution. This is usually written as $Q \sim \chi^2_k$

The expression for P_e can be rewritten as $P_e = \frac{v^2}{nR} \sum (v_i')^2$ where v_i' is the normalised sample voltage such that its variance is 1, or $P_e' = \sum (v_i')^2$, and we can say that $P_e' \sim \chi^2_k$ (for $k=n-1$).

2. Uncertainty

Since $P_e' \sim \chi^2_n$, the critical points of the chi-square distribution can be used to find confidence limits for noise measurements based on an effective sample size.

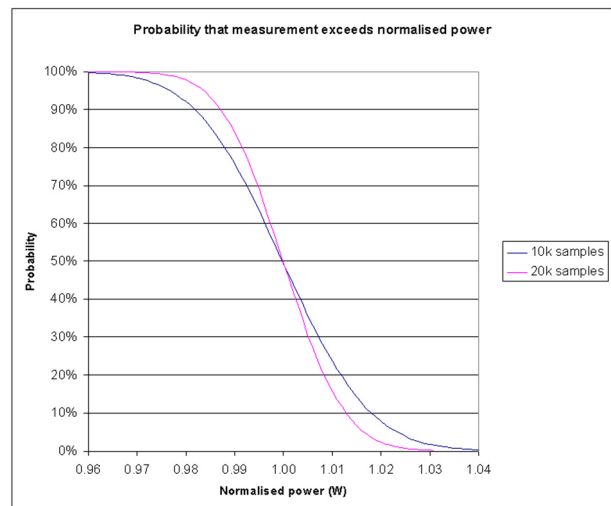


Figure 1: Probability that measurement exceeds normalised power.

Remembering that the measurement process is only an estimate of the true noise power, and that successive sample sets are likely to produce similar, but different estimates. Figure 1 shows the probability that a measurement exceeds a given normalised power level for measurements based on 10,000 and 20,000 samples. The curves tell us that 90% of measurements based on 20,000 samples will indicate more than 0.987W (normalised power). Similarly, 10% of measurements based on 20,000 samples will indicate more than 1.013W, and so 90%-10% or 80% of measurements will be between 0.987W and 1.013W of the correct power. We can state that the sampling uncertainty is $\pm 1.3\%$ to a confidence level of 80%.

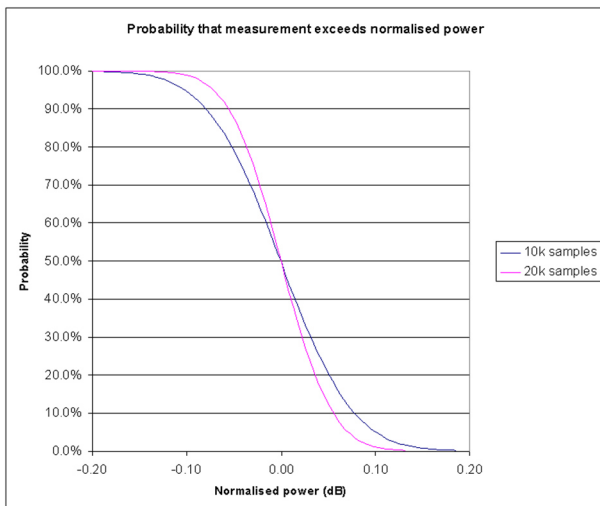


Figure 2: Probability that measurement exceeds normalised power.

Looking at Figure 2 which shows the probability that a measurement exceeds a given normalised power level for measurements based on 10,000 and 20,000 samples. The curves tell us that 90% of measurements based on 20,000 samples will indicate more than -0.06dB (normalised power). Similarly, 10% of measurements based on 20,000 samples will indicate more than 0.06dB, and so 90%-10% or 80% of measurements will be between -0.06dB and 0.06dB of the correct power. We can state that the sampling uncertainty is $\pm 0.06\text{dB}$ to a confidence level of 80%.

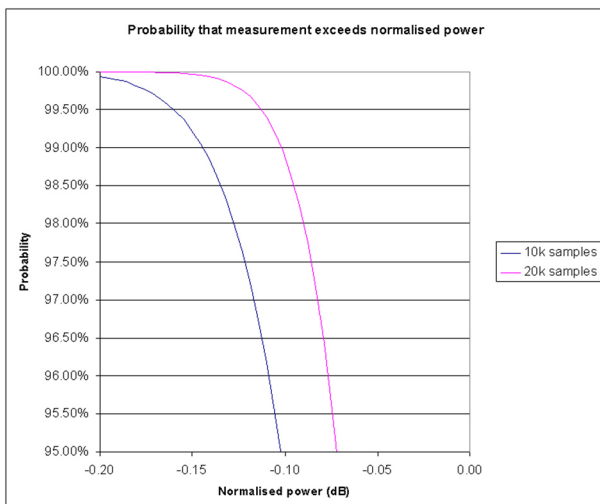


Figure 3: Probability that measurement exceeds normalised power.

Figure 3 expands part of Figure 2. Looking at the curves, the 97.5% confidence limit for 20k samples is -0.086dB, and for 10k samples is -0.12dB. These results are summarised in Table 1.

Table 1

Sample set size	Sampling uncertainty (dB)
10k	± 0.120
20k	± 0.086

Increasing sample set size reduces sampling uncertainty.

3. Effective sample set size

The Nyquist–Shannon sampling theorem states that exact reconstruction of a continuous-time baseband signal from its samples is possible if the signal is bandlimited and the sampling frequency is greater than twice the signal bandwidth.

This implies that a sample set with sampling frequency higher than twice the highest frequency, the Nyquist Rate, contains no more information than one with sampling frequency equal to the Nyquist Rate.

Further, that translation of a limited bandwidth noise to another frequency range does not change the noise power (mean and variance) of the signal, and that provided that the sampling rate is greater than the Nyquist Rate (so as to prevent aliasing), that the sample set contains no more information than one with sampling frequency equal to twice the bandwidth.

Applying this to measurement of noise:

- a digital process to sample a bandlimited noise waveform must sample the waveform at a rate higher than twice the highest frequency; and
- the effective sample rate for determining the distribution of measured power is twice the bandwidth.

The effective sample set size is $\text{Bandwidth} \times \text{IntegrationTime} / 2$ provided that the sample rate is at least twice the highest frequency.

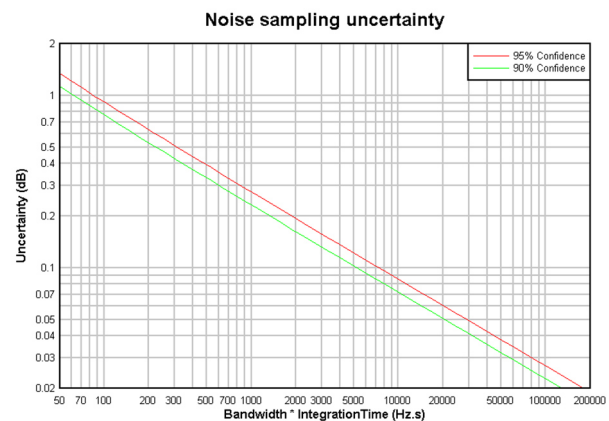


Figure 4: Noise sampling uncertainty.

Figure 4 shows the expected sampling uncertainty against the $\text{Bandwidth} \times \text{IntegrationTime}$ product. This indicates that if you wanted to reduce sampling uncertainty to 0.1dB at a 95% confidence level, you need a $\text{Bandwidth} \times \text{IntegrationTime}$ product of greater than 7,400. With a 2kHz wide receiver, and IntegrationTime of 7400/2000 or 3.7s would be required.

A handy online Noise Measurement Uncertainty Calculator is at <http://owenduffy.net/calc/nmuc.htm>.

4. Experimental results

An experiment was conducted to compare the spread of a series of measurements with predictions. Groups of 30 measurements were made of a receiver with effective noise bandwidth of 1500Hz (250Hz to 1750Hz) and with IntegrationTime of 0.15s, 1.5s, and 15s.

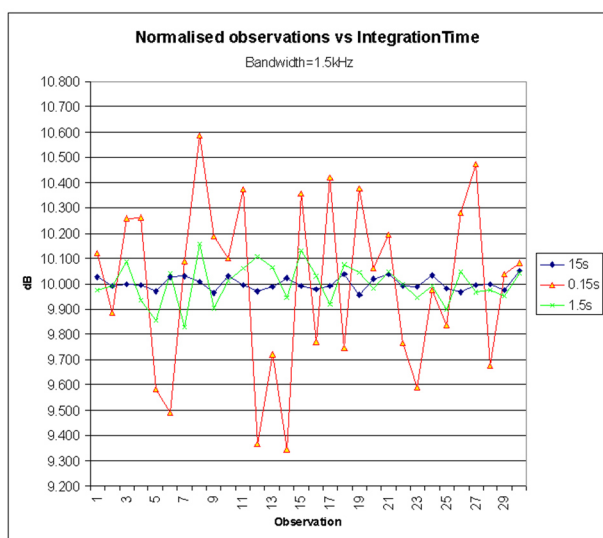


Figure 5: Experimental measurements.

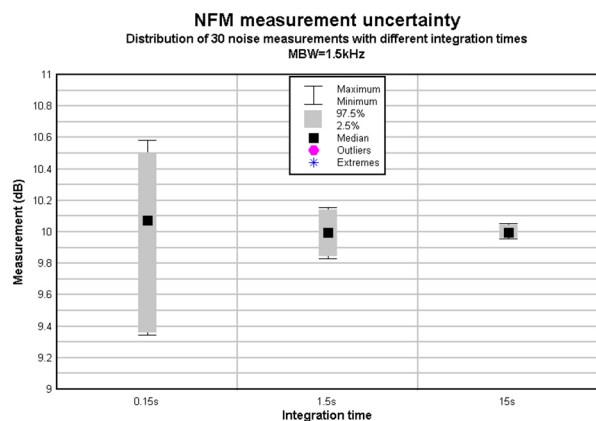


Figure 6: Experimental results.

Figure 6 shows the distribution of the measurements in each of the three groups.

Table 2

Integration Time (s)	Measurements	Predicted sampling uncertainty (dB)	Actual 95% interval (dB)
0.15	30	±0.578	±0.572
1.5	30	±0.183	±0.146
15	30	±0.058	±0.040

Table 2 shows the predicted and experimental results.

Table 3

Integration Time (s)	Measurements	Predicted sampling uncertainty (dB)	Actual 95% interval (dB)
0.15	997	±0.578	±0.570

Table 3 shows the predicted and experimental results from a larger test.

5. Agilent PN 85719A-1

A relevant product note by Agilent has come to hand. PN 85719A-1 entitled "Maximizing Accuracy in Noise Figure Measurements" gives an expression for calculating the expected variation in noise measurements: $variation[dB] = 10 \log(1 + 3(t \cdot BW)^{-0.5})$ for $t \gg BW$.

The expression is held to be good for $t \gg BW$, but what does that mean exactly?

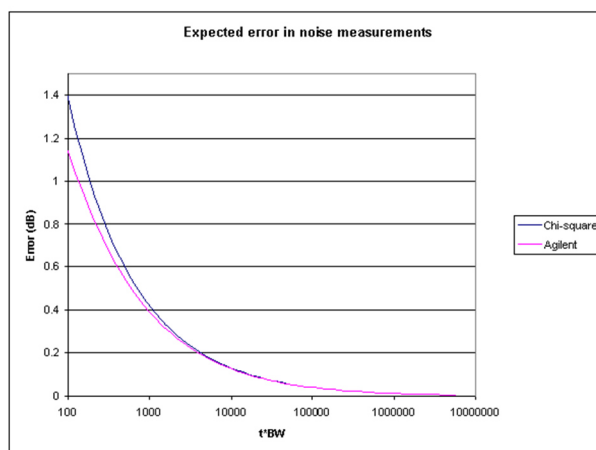


Figure 7: Comparison of Chi-square estimate and Agilent's estimate.

Figure 7 is a comparison of the chi-square estimate at confidence level 99.73% (equivalent to Normal distribution 3σ limits) with Agilent's expression. The difference is less than 0.01dB for $t*BW > 2700$. Agilent's expression would appear to be an approximation of the chi-square estimate, and is reasonably accurate for $t*BW$ greater than about 2000.

Agilent's approach may be based on a Normal approximation of χ^2_k for large k , but it appears to be a poor estimate for smaller k .

6. Normal approximation

As $k \rightarrow \infty$, $\frac{(\chi^2_k - k)}{\sqrt{2k}} \xrightarrow{d} N(0,1)$, so for very large k ($t*BW$), greater than say 2,000, the normal approximation may provide an adequate approximation. It can be seen from Figure 4 that for 2σ uncertainty expectation due to

sampling alone below 0.25dB, the more exact chi-square calculation is better.

7. Glossary

Term	Meaning
Aliasing	Distortion caused when the sample rate of a function (f_s) is less than twice the highest frequency value of the input waveform or function. The frequencies above ($f_s/2$) will be folded back into the lower frequencies.
Baseband	signals that contain a band of frequencies from zero and have a bandwidth of highest signal frequency.
dB	decibel - power ratio
dBm	decibels - power wrt 1 mW
Nyquist frequency	Half the sampling frequency of a discrete signal processing system ($f_s/2$)
Nyquist Rate	The minimum sampling rate required to avoid aliasing, equal to twice the highest frequency contained within the signal (f_s)

8. Links

Noise Measurement Uncertainty Calculator

<http://owenduffy.net/calc/nmuc.htm>.

FSM (Field Strength Meter) software

<http://owenduffy.net/software/fsm/index.htm>.

NFM (Noise Figure Meter) software

<http://owenduffy.net/software/nfm/index.htm>.

9. Changes

Version	Date	Description
1.01	28/06/2007	Initial.
1.02	24/03/2009	Added Agilent update.
1.03	12/03/2016	Converted to Word, URL updates.
